

GMAT Arithmetic: Challenge (Excerpt)

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June 2009

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1 Introduction

This document contains nothing but difficult GMAT Arithmetic questions—100 of them, to be exact. The term "arithmetic" doesn't sound particularly hard: it conjures up memories of multiplication tables and long division from elementary school. However, simple concepts are the ones the GMAT most likes to test, but in devilish ways. In fact, some of the trickiest questions on the test are based on concepts such as remainders and averages.

As in all of my GMAT preparation resources, you'll find these questions indexed by difficulty. That doesn't mean you should skip straight to the hardest questions, or even that you should start with the easier ones. On the GMAT itself, questions won't come labeled with their difficulty level, and despite the intent of the adaptive algorithm, they won't be precisely consistent in terms of difficulty either. Each question presents its own unique challenges, and the sooner you get accustomed to changing gears with every single question, the more time you'll have to prepare for that particular challenge of the exam.

For further, more specific practice, I have produced several other resources that may help you. Another one of my 100-question practice sets, "Statistics and Sets," focuses entirely on statistical measures such as average, mean, median, range, and standard deviation, along with set-related issues, such as Venn diagrams. I've also created a 100-question set called "Exponents and Roots," which covers exactly that material, including dozens of questions that force you to master every last exponent-related rule the GMAT will test you on.

Also, The GMAT Math Bible has several chapters (along with focused practice) on Arithmetic and related issues, including individual chapters on fractions, decimals, exponents, roots, ratios, percents, rate, mixture, interest, average, weighted average, statistics, overlapping sets, sequences, and more.

If you find yourself having problems with only the most difficult questions, you might try my "Extreme Challenge" set, which contains only 720 and higher level questions, many of which are Arithmetic-related.

As far as strategy is concerned, there are dozens of articles at GMAT HACKS to help you with your strategic approach to Arithmetic questions. Most importantly, you should make sure you understand every practice problem you do. It doesn't matter if you get it right the first time—what matters is whether you'll get it right the next time you see it, because the next time you see it could be on the GMAT.

With that in mind, carefully analyze the explanations. Redo questions that took you too long the first time around. Review questions over multiple sessions, rather than cramming for eight hours straight each Saturday. These basic study skills may not feel like the key to GMAT preparation, but they are the difference between those people who reach their score goals and those who never do.

Enough talking; there are 100 Arithmetic questions waiting inside. Get to work!

2 Difficulty Levels

In general, the level 5 questions in this guide are 560- to 620-level questions. The level 6 questions represent a broad range of difficulty from about 620 to 720, while the level 7 questions are higher still.

Moderately Difficult (5)

PS

3, 5, 14, 15, 21, 24, 28, 30, 31, 35, 43, 49, 51, 54, 58, 63, 66

DS

74, 76, 79, 81, 83, 86, 87, 89, 93, 99, 100

Difficult (6)

PS

2, 4, 6, 9, 10, 12, 13, 16, 17, 18, 20, 22, 23, 25, 26, 27, 29, 34, 36, 37, 39, 40,
41, 42, 44, 45, 46, 47, 48, 50, 52, 53, 55, 57, 59, 60, 61, 62, 64, 65, 69

DS

71, 73, 75, 77, 78, 80, 82, 90, 92, 94, 95, 96, 97, 98

Very Difficult (7)

PS

1, 7, 8, 11, 19, 32, 33, 38, 56, 67, 68, 70

DS

72, 84, 85, 88, 91

3 Problem Solving

Note: this guide contains both an answer key (so you can quickly check your answers) and full explanations.

68. If $y^2 = 2$, what is the value of $\frac{2^{(x-y)(x+y)}}{2^{(x-1)(x+1)}}$?
- (A) -2
 - (B) $-\frac{1}{2}$
 - (C) $\frac{1}{2}$
 - (D) 2
 - (E) 4
69. In a certain lottery drawing, two balls are selected at random from a container with 100 balls, numbered from 1 to 100, inclusive. If the winner of the lottery is awarded a cash prize in the amount of \$1,000 times the product of the numbers on the two selected balls, which of the following is a possible amount of the cash prize given to the winner?
- (A) $\$9.85 \times 10^6$
 - (B) $\$9.90 \times 10^6$
 - (C) $\$1.00 \times 10^7$
 - (D) $\$1.05 \times 10^7$
 - (E) $\$9.90 \times 10^7$
70. John's car dealership contains m cars, 20% of which are minivans and 80% are sedans. Kevin's car dealership contains n cars, 40% of which are minivans and 60% are trucks. Larry's car dealership contains p cars, 50% of which are minivans and 50% of which are convertibles. If 25% of the $m + n + p$ cars contained at the three dealerships are minivans, what is m in terms of n and p ?
- (A) $n + 3p$
 - (B) $3n + 5p$
 - (C) $4n + 5p$
 - (D) $\frac{n+5p}{2}$
 - (E) $\frac{4n+5p}{3}$

4 Data Sufficiency

For all Data Sufficiency questions, the answer choices are as follows:

- (A) Statement (1) ALONE is sufficient, but statement (2) alone is not sufficient.
 - (B) Statement (2) ALONE is sufficient, but statement (1) alone is not sufficient.
 - (C) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
 - (D) EACH statement ALONE is sufficient.
 - (E) Statements (1) and (2) TOGETHER are NOT sufficient.
71. If $y = 0.abcd$, where a , b , c , and d each represent a nonzero digit of y , what is the value of y ?
- (1) $a = d^2$ and $b = c^2$
 - (2) $a > b$
72. If a and b are positive integers, what is the remainder when 4^{2a+1+b} is divided by 10?
- (1) $a = 1$
 - (2) $b = 2$
73. 35 percent of the books in Richard's collection are novels and 60 percent of the books are paperbacks. If 20 percent of the paperbacks in Richard's collection are novels published before 1940, how many of the paperbacks in Richard's collection are novels published before 1940.
- (1) Non-fiction books make up exactly half of Richard's collection.
 - (2) 100 of the books in Richard's collection are non-fiction.
74. The number N is $2,3H4$ the ten's digit being represented by H . What is the value of H ?
- (1) N is divisible by 9.
 - (2) N is divisible by 4.
75. If x is less than 75 percent of y , is x less than 75 ?
- (1) $y > 20$
 - (2) $x + 25 = y$

4. DATA SUFFICIENCY

76. A Town T has 40,000 residents, 25 percent of whom are age 60 or older. What percent of the residents are female?
- (1) The number of female residents of Town T age 60 or older is 30 percent of the number of female residents who are not 60 or older.
 - (2) 37 percent of the residents of Town T are males who are not 60 or older.
77. For a certain set of n numbers, where $n > 2$, is the average (arithmetic mean) equal to the median?
- (1) The n numbers are positive, consecutive even integers.
 - (2) The average of the n numbers is equal to the average of the largest and smallest numbers in the set.

5 Answer Key

For full explanations, see the next section.

- 68. C
- 69. B
- 70. B
- 71. E
- 72. B
- 73. C
- 74. C
- 75. B
- 76. C
- 77. A

6 Explanations

For a quick-reference answer key, see the previous section.

68. C

Explanation: The first step is to subtract the exponents from each other. Ignoring for a moment that they are an exponent:

$$\begin{aligned}(x - y)(x + y) - (x - 1)(x + 1) \\ x^2 - y^2 - (x^2 - 1) \\ -y^2 + 1\end{aligned}$$

We know that $y^2 = 2$, so $-y^2 + 1 = -2 + 1 = -1$.

Finally, if the exponent is -1, the answer is $2^{-1} = \frac{1}{2}$, choice (C).

69. B

Explanation: Since 9.9 appears in two of the choices, that's a good place to start. The cash prize could end up being 9.9 plus some number of zeroes if the two balls drawn were 99 and 100. Then, the product would be 9,900, and the cash prize would be 9,900 times \$1,000. To translate that to scientific notation:

$9.9 \times 10^3 \times 1 \times 10^3 = 9.9 \times 10^6$, choice (B). We're only looking for a possible answer, and since (B) is clearly possible, we're done.

70. B

Explanation: Ignore all the details of the types of cars—the key thing is that all of the mixes of cars involve some number of minivans. Write out an equation representing the number of minivans in each lot:

$$0.2m + 0.4n + 0.5p = 0.25(m + n + p)$$

In each term, the percent of minivans is multiplied by the total number of cars to give the number of minivans. The total number of minivans in each of the three dealerships is equal to 25% of the total number of cars in the three dealerships.

Solve for m :

$$0.2m + 0.4n + 0.5p = 0.25m + 0.25n + 0.25p$$

$$0.15n + 0.25p = 0.05m$$

$$15n + 25p = 5m$$

$$3n + 5p = m$$

Choice (B) is correct.

71. E

Explanation: Statement (1) is insufficient: in both pairs of variables, they could be 4 and 2, respectively, 9 and 3, respectively, or even 1 and 1.

Statement (2) is insufficient: it gives no information about two of the four variables.

Taken together, we still don't have enough information. It means that a and d can't be 1 and 1, respectively, and that b and c can't be 9 and 3, respectively, but it leaves three sets of possible values for the four variables:

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0.2114

0.9113

0.9423

Choice (E) is correct.

72. B

Explanation: Essentially, we're looking for the units digit of 4^{2a+1-b} . When 4 is raised to an integer power, the units digit is always either 4 (if the power is odd: 4, 64, etc.), or 6 (if the power is even: 16, 256, etc.). So what we really need to know is whether the power is even or odd.

Statement (1) is insufficient. $2a + 1 + b$ is always *even* + 1 + b regardless of the value of a , since that value is multiplied by 2.

Statement (2) is sufficient: $2a + 1 + b = \text{even} + 1 + 2 = \text{even} + 3 = \text{odd}$, so no matter what the value of a , the power is odd, the the remainder is 4. Choice (B) is correct.

73. C

Explanation: The question gives us several pieces of information:

$$0.35b = n$$

$$0.35b = p$$

$$0.2p = \text{novelsBefore1940}$$

All of these are in terms of percents, so if we want a number, we need a number.

Statement (1) doesn't give us another number: it's another ratio, so we can't find an actual amount.

Statement (2) gives us a number, but it doesn't relate to any of the percents. We don't know anything about the size of the collection on the basis of this information.

Combining the two statements, we have enough information. If 100 non-fiction books constitute half of the collection, the total number of books is 200. With that, we can find the number of paperbacks, and from there, the number of paperbacks that are novels published before 1940. Choice (C) is correct.

74. C

Explanation: Statement (1) is insufficient. If a number is divisible by 9, the digits must sum to 9. $2 + 3 + 4 = 9$, so $2 + 3 + H + 4$ will sum to a multiple of 9 if $H = 0$ or if $H = 9$.

Statement (2) is also insufficient. If a number is divisible by 4, the number formed by the last two digits must be a multiple of 4. There are many possibilities for H here: it could be 0, 2, 4, 6, or 8.

Taken together, the statements are sufficient. (1) gives only two possibilities for H , and (2) only allows for one of them. $H = 0$, so choice (C) is correct.

75. B

Explanation: The question gives us an inequality:

$$x < 0.75y$$

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And the question:

$$x < 75 ?$$

Statement (1) is insufficient. If $y > 20$, $0.75y > 15$, so the question is: "is something less than something greater than 15 less than 75?" Without knowing more about the range of y , we can't answer the question.

Statement (2) is sufficient. We can substitute the equation into the equation given in the question:

$$x < 0.75y$$

$$x < 0.75(x + 25)$$

$$x < 0.75x + \frac{3}{4}(25)$$

$$0.25x < \frac{3}{4}(25)$$

$$x < 3(25)$$

$$x < 75$$

Combining our pieces of given information directly answers the question.

76. C

Explanation: Statement (1) is insufficient. It gives us the relationship between the number of women under 60 to those at least 60. By extension, it gives us the relationships between those subgroups and the total number of women, but since we don't have numbers or percents for any of those three, it's not good enough.

Statement (2) is also insufficient. Since we know 25 percent of total residents are 60 or older, 75 percent are under 60. If 37 percent of the total are males under 60, that leaves 38 percent who are females under 60. Still, we don't know how many females are 60 or older.

Taken together, the statements are sufficient. (2) tells us what percent of the total are females under 60, and (1) gives us the relationship between that number and the total number of female residents, which is what we're looking for. (C) is correct.

77. A

Explanation: The mean and median of a set will always be equal if the set consists of consecutive (equally-spaced) numbers. There are other times when the mean and median are equal, but the GMAT likes to test the first concept.

Statement (1) is sufficient: if the terms are consecutive evens, they are equally spaced, so the mean and the median are the same.

Statement (2) is insufficient. That the mean of the set is equal to the mean of the largest and smallest numbers doesn't tell us anything about how the inner numbers are arranged; there are far too many variables concerning how the numbers are spaced to know how the median relates to the mean. Choice (A) is correct.